Correlation functions for quantum one-dimensional systems

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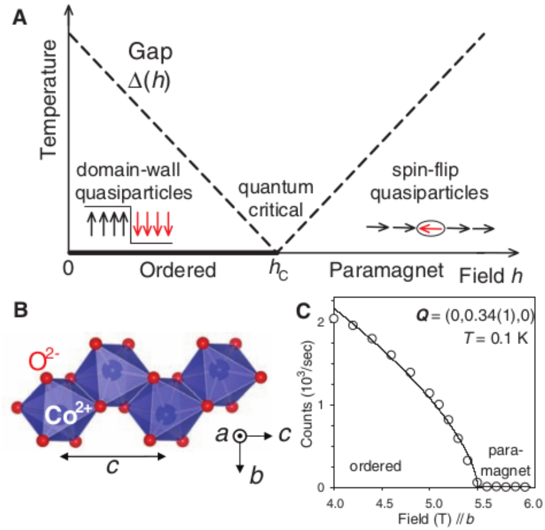
Motivation

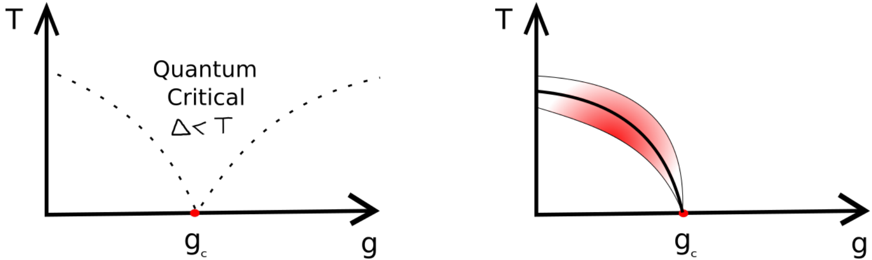
Physical realizations of 1D quantum Ising model



Coldea, R., et al., Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry (2010) Science 327, 177.

Quantum phase transitions

$$H = H(g)$$



S. Sachdev, Quantum Phase Transitions, (Cambridge University Press, 2011)

Main problem

Hamiltonian of quantum Ising model

$$\mathcal{H} = -\frac{J}{2}\sum_{m=1}^{N}\left(\sigma_m^x \sigma_{m+1}^x + h\sigma_m^z\right),$$

h – external field.

Correlation function

$$G(m) = \langle \sigma_0^x \sigma_m^x \rangle \sim A e^{-\frac{m}{\xi}}.$$

Ising field theory

Scaling limit $J \to \infty, \ell \to 0, h \to 1$

Introduce ℓ – lattice spacing.

$$p = P\ell, \qquad n = \frac{x}{\ell}$$

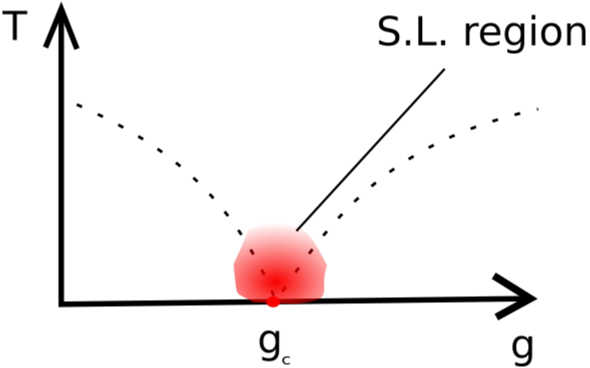
At $\ell \to 0$ dispersion relation becomes relativistic!

$$\mathcal{E}^2(p) = J(h - \cos p)^2 + \gamma^2 \sin^2 p \ \to \ \Delta^2 + P^2 C^2.$$

Gap (mass) and 'speed of light':

$$\Delta = J(1 - h), \qquad C = J\ell.$$

Scaling limit $J \to \infty, \ell \to 0, h \to 1$



Ising field theory

Fermionic Hamiltonian:

$$\mathcal{H}_{XY} = -\frac{J}{2}\sum_j \left(a^\dagger_{j+1}a_j + a^\dagger_j a_{j+1} + \gamma a^\dagger_j a^\dagger_{j+1} + \gamma a_j a_{j+1} + h - 2h a_j a^\dagger_j \right).$$

Introduce rescaled field

$$\Psi(x) = \frac{1}{\sqrt{\ell}} a_j.$$

Field Hamiltonian

$$\mathcal{H}_f = E_0 - \int dx \left(\frac{c}{2}(\Psi^\dagger \partial_x \Psi^\dagger - \Psi \partial_x \Psi) + \Delta \Psi^\dagger \Psi \right).$$

S. Sachdev, Quantum Phase Transitions, (Cambridge University Press, 2011)

Correlation functions at $T = 0$

Equal-time correlation function

$$\langle 0|\sigma(0)\sigma(x)|0\rangle \sim Ae^{-\frac{x}{\xi}}.$$

Time

$$\sigma(x,t) = e^{\mathrm{i}tH}\sigma(x)e^{-\mathrm{i}tH}.$$

Time dependent correlation function

$$\langle 0|\sigma(0,0)\sigma(x,t)|0\rangle \sim K_0(\Delta\sqrt{x^2 - c^2t^2}).$$

S. Sachdev, A.P. Young, Low temperature relaxational dynamics of the Ising chain in a transverse field, (1997) arxiv.org/abs/cond-mat/9609185

Correlation functions at $T \neq 0$

Equal-time correlation function

$$G(x) = \langle \sigma(0)\sigma(x) \rangle = \frac{tr(\sigma(0)\sigma(x)e^{-\beta H})}{tr(e^{-\beta H})} \sim Ae^{-\frac{x}{\xi}}.$$

Time dependent correlation function

$$G(x,t) = \langle \sigma(0,0)\sigma(x,t) \rangle = \frac{tr(\sigma(0)e^{\mathrm{i}tH}\sigma(x)e^{-\mathrm{i}tH}e^{-\beta H})}{tr(e^{-\beta H})} \sim Be^{-\frac{x}{\xi}}e^{R(t,x)}.$$

B.L. Altshuler, R.M. Konik, A.M. Tsvelik, Low temperature correlation functions in integrable models: derivation of the large distance and time asymptotics from the form factor expansion, (2006)
arxiv.org/abs/cond-mat/0508618

Correlation functions at $T \neq 0$

In the thermodynamic limit

$$G(m) = \det\left(1 + \hat{W}\right),$$

where

$$W(q,p) = \frac{e^{2\pi i\nu(p)} - 1}{2\pi} \frac{\sin(m\frac{p-q}{2})}{\sin(\frac{p-q}{2})},$$

$$e^{2\pi i\nu(p)} = -e^{i(\theta(p)-p)} \tanh\frac{\beta\mathcal{E}(p)}{2}.$$

O. Gamayun, N. Iorgov, Yu. Zhuravlev, Effective free-fermionic form factors and the XY spin chain, (2021) arxiv.org/abs/2012.02079

Results

Prefactor on the lattice

$$\log A = -\frac{1}{2}\int_{-\pi}^{\pi} dp \int_{-\pi}^{\pi} dq \left[\frac{\nu(q)-\nu(p)-(q-p)/2\pi}{2\sin\frac{p-q}{2}}\right]^2,$$

де $\nu(p) = \frac{\theta(p)}{2\pi} + \frac{1}{2i\pi}\log\tanh\frac{\beta\mathcal{E}(p)}{2}$.

O. Gamayun, N. Iorgov, Yu. Zhuravlev, Effective free-fermionic form factors and the XY spin chain, (2021) arxiv.org/abs/2012.02079

in the scaling limit

$$A = \Delta^{1/4}\exp\frac{1}{8\pi^2}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx \frac{(\mu(y)-\mu(x))^2}{(y-x)^2},$$

$\mu(x) = \log\tanh\frac{\beta\Delta\sqrt{1+x^2}}{2}$

Summary and discussion

Result:

Scaling limit for **asymptotics** of correlation function in quantum Ising model at nonzero temperature from the Fredholm determinant representation.

Open questions:

Scaling limit for correlation function in quantum Ising model, taking into account time.

Thank you for your attention!