Large time and long distance asymptotics of the thermal correlators of the impenetrable anyonic lattice gas

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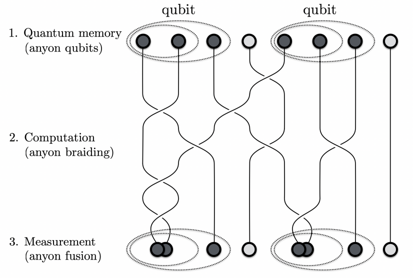
Motivation

- Topological quantum computing



J. K. Pachos, Cambridge University Press, Cambridge, ISBN 9780511792908 (2012)

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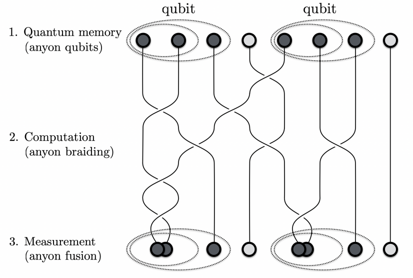

J. K. Pachos, Cambridge University Press, Cambridge, ISBN 9780511792908 (2012)

- Systems of ultracold atoms

T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, Nature Communications 2, 361 (2011)

Bosons	Fermions
$[a_j; a_m^y] = \ _{jm}$	$f a_j; a_m^y g = \ _{jm}$
$[a_j; a_m] = 0$	$f a_j; a_m g = 0$
$[a_j^y; a_m^y] = 0$	$f a_j^y; a_m^y g = 0$

Bosons

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Fermions

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Anyons

$$a_j a_m^y = \quad _{jm} \quad e^{\ i \quad (j \quad m)} a_m^y a_j$$
$$a_j a_m = \quad e^{i \quad (j \quad m)} a_m a_j$$
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Bosons | Fermions

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Anyons

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$$a_j^y a_m^y = \quad e^{i \quad (j \;\; m)} a_m^y a_j^y$$

$(m) = m{=}jmj, \quad (0) = 0,$

$2\ [0; 1]$ – statistics parameter.

Anyonic model

$$H = \sum_{j=1}^{L} \frac{1}{2}(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) + h \sum_{j=1}^{L} a_j^\dagger a_j;$$

$$a_{L+1} = a_L; \qquad a_{L+1}^\dagger = a_L^\dagger:$$

L – number of lattice sites

h – chemical potential

Anyonic model

Two-point correlation function ($L \to \infty$):

$$G(x;t) = \frac{\mathrm{Tr}[e^{-\beta H} a^{\dagger}_{x+1}(t) a_1(0)]}{\mathrm{Tr}[e^{-\beta H}]},$$

where $\beta = 1/T$; $a^{\dagger}_x(t) = e^{iHt} a^{\dagger}_x e^{-iHt}$.

Anyonic model

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- For large x and t it is difficult to compute Fredholm determinants numerically;

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- For large x and t it is difficult to compute Fredholm determinants numerically;
- One need to find more effective ways to study asymptotics.

Effective form factor approach

To specify the effective form factor we require two smooth periodic functions (k), $g(k)$. Here L is regarded as a system size.

- The first one is called the effective phase shift and defines the *shifted* set of momenta as solutions of

$$e^{ikL} = e^{\ 2\ i\ (k)}; \quad e^{iqL} = 1:$$

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- The second function is in the definition of effective form factors

$$|\langle \mathbf{k}|\mathbf{q}^{(a)}\rangle|^2 = L^{1\ \ 2L} \prod_{j=1}^{L} \frac{e^{g(k_j)\ \ g(q_j)} \sin^2\ \ (k_j)}{1 + \frac{2}{L}\ \ '(k_j)} e^{g(q_a)} \det{}^2 D^a,$$

where

$$\det D^a = \begin{vmatrix} \cot\frac{k_1\ \ q_1}{2} & \dots & \cot\frac{k_L\ \ q_1}{2} \\ \vdots & \ddots & \vdots \\ \cot\frac{k_1\ \ q_L}{2} & \dots & \cot\frac{k_L\ \ q_L}{2} \\ 1 & \dots & 1 \end{vmatrix},$$

$$\mathbf{q}^{(a)} = \{q_1, \dots, q_{a\ 1}, q_{a+1}, \dots, q_L\}, \quad a = 1, \dots, L.$$

Effective form factor approach

The tau (correlation) function is defined as series over these form factors

$$\tau(x,t) = \sum_{\mathbf{q}^a} |\langle \mathbf{k}|\mathbf{q}^a\rangle|^2 e^{-ix(P(\mathbf{k})-P(\mathbf{q}^a))+it(E(\mathbf{k})-E(\mathbf{q}^a))},$$

where

$$P(\mathbf{q}) = \sum_{q\in\mathbf{q}} q, \quad E(\mathbf{q}) = \sum_{q\in\mathbf{q}} \varepsilon(q), \quad \varepsilon(q) = h - \cos q.$$

Effective form factor approach

$$(x;t) = \text{Fredholm determinants} \quad g(q) \quad x \quad t \quad (q) \quad h$$

Effective model
$g(\mathrm{q}) \quad (\mathrm{q}) \quad \mathrm{h}$

Anyonic model
h

Effective form factor approach

$$(x;t) = \begin{array}{c}\text{Fredholm}\\ \text{determinants}\end{array} \quad g(q) \quad {}^{x \quad t}(q) \quad h$$

Effective model		Anyonic model
$g(\mathrm{q})$ (q) h	$\overset{G(x;t)=\ (x;t)}{(=========)}$	h

Effective form factor approach

$(x;t)$ = Fredholm determinants $g(q)$ x (q) t h

Effective model
$g(q)$ (q) h

$G(x;t) = \ (x;t)$

Anyonic model
h

Representation $(x;t)$ as an integral

Effective form factor approach

$$(x;t) = \text{Fredholm determinants} \quad x \quad t \quad g(q) \quad (q) \quad h$$

Effective model

g(q) (q) h

$G(x;t) = (x;t)$

⟺

Anyonic model

h

⇓

⇓

Representation $(x;t)$ as an integral

⟺

Asymptotics for anyonic model

Effective form factor approach

$(x;t) =$ Fredholm determinants $x \quad t$ $g(q) \quad (q) \quad h$

Effective model
$g(\mathrm{q}) \quad (\mathrm{q}) \quad \mathrm{h}$

$G(x;t) = \ (x;t)$ (========)

Anyonic model
h

⇓

Representation $(x;t)$ **as an integral**

(========)

Asymptotics for anyonic model

O. Gamayun, N. Iorgov, and Y. Zhuravlev, Effective free-fermionic form factors and the XY spin chain, SciPostPhys. 10, 70 (2021).

Results

Space-like region ($x > t$), $x \quad 1$, $t \quad 1$

$$G(x;t) \quad C_2 K(x;t) e^{\, x \log z_0 + \frac{t}{}(1 \quad)}$$

where $K(x;t)$ and z_0 are given by

$$K(x;t) = Z^2[\ ] e^{ix \int (q) dq}; \qquad (q) = \ _+(q);$$

$$(q) = \frac{1}{2\ i} \log \ 1 + n_F(q)(e^{\ i} \qquad 1) \ ;$$

$$n_F(q) = \frac{1}{e^{\ ''(q)} + 1}; \qquad ''(q) = h \quad \cos q;$$

$$z_0 = h_0 + \sqrt{h_0^2 \quad 1}; \quad h_0 = h + \frac{i}{}(1 \qquad):$$

The prefactors $Z^2[\ ]$ and C_2 are constants for fixed x/t.

Results

Time-like region ($x < t$), x 1, t 1

$$G(x;t) \quad R_1 t \quad {}^2_1 \quad {}^2_2 e^{i \int (x \quad t\,\varepsilon'(q))\ (q)dq}$$

$$a_1 e^{ixq_1 + it\,\varepsilon(q_1)} \over t^{\frac{1}{2} + \ 1} \quad + \quad {a_2 e^{ixq_2 + it\,\varepsilon(q_2)} \over t^{\frac{1}{2} + \ 2}} \quad !$$

The critical momenta q_1 and q_2 are defined by

$$q_1 = \arcsin(x{=}t); \qquad q_2 = \qquad \arcsin(x{=}t);$$

the effective phase shift (q) is piecewise function

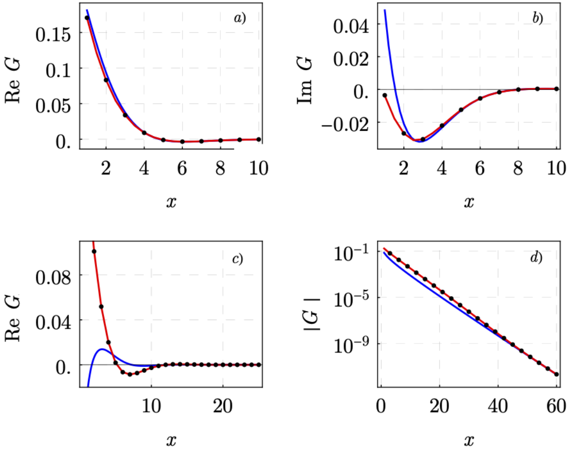
$$(q) = \begin{cases} {}_{+}(q) & \text{if} \qquad < q < q_1 \text{ or } q_2 < q \qquad ; \\ (q) & \text{if } q_1 < q < q_2; \end{cases}$$

and $_1$ and $_2$ are the magnitudes of jumps of (q) at critical momenta

$$_1 = \quad (q_1) \quad {}_{+}(q_1); \qquad _2 = {}_{+}(q_2) \quad (q_2):$$

Results

Asymptotics for space-like region



Black dots – numerical values of Fredholm determinants, blue lines – asymptotics.
Panels *a*) and *b*) correspond to *x*=*t* = 2:5, panels *c*) and *d*) correspond to *x*=*t* = 1:3.
= 0:6, *h* = 0:7, = 2:3.

Y. Zhuravlev, E. Naichuk, N. Iorgov and O. Gamayun, Phys. Rev. B 105, 085145 (2022)

Conclusions

- Asymptotic behaviour of the correlation function at large time and long distance in both space-like and time-like regions was derive;
- It was found that on top of the exponential decay the additional power factor appears in the time-like region.

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- It was found that on top of the exponential decay the additional power factor appears in the time-like region.

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Thank you for your attention!